Dynamic Group Diffie-Hellman Key Exchange under Standard Assumptions

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OUTLINE

- Motivation and Previous Work
- Communications and Security Model
- A Secure Group DH Protocol
- Standard Assumptions
- Security Theorem and its Proof
- * Conclusion

Motivation

- An increasing number of distributed applications need to communicate within groups, e.g.
 - collaboration and videoconferencing tools
 - replicated servers and distributed computations
- An increasing number of applications have security requirements
 - privacy of data
 - protection from hackers, viruses and trojan horses
- Group communication must address security needs

The Problem

- Group Characteristics
 - group relatively small (<100 members), dynamic
 - members have similar computing power
 - no centralized server
- Goals for Group Key Exchange
 - * Authenticated Key Exchange (AKE) implicit authentication: only the intended partners get sk semantic security: no information leaks about sk
 - Mutual Authentication (MA)
 key confirmation mechanism

Prior Work

- * "Provably Authenticated Group DH Key Exchange: The Dynamic Case", [A'01]
 - model of computation in the Bellare-Rogaway style
 adversary controls the network
 adversary's interacts with players via oracle queries
 - * a group DH key exchange protocol SETUP, JOIN, REMOVE algorithms
 - * security proof

 sequential executions only
 ideal-hash assumption

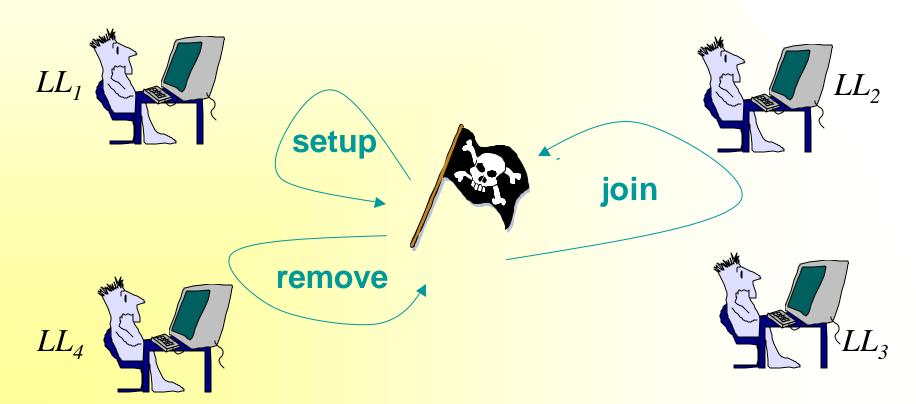
Model of Communication

- * A multicast group consisting of a set of players
 - each player holds a long-lived key (LL)
 - each player holds ephemeral internal keying material

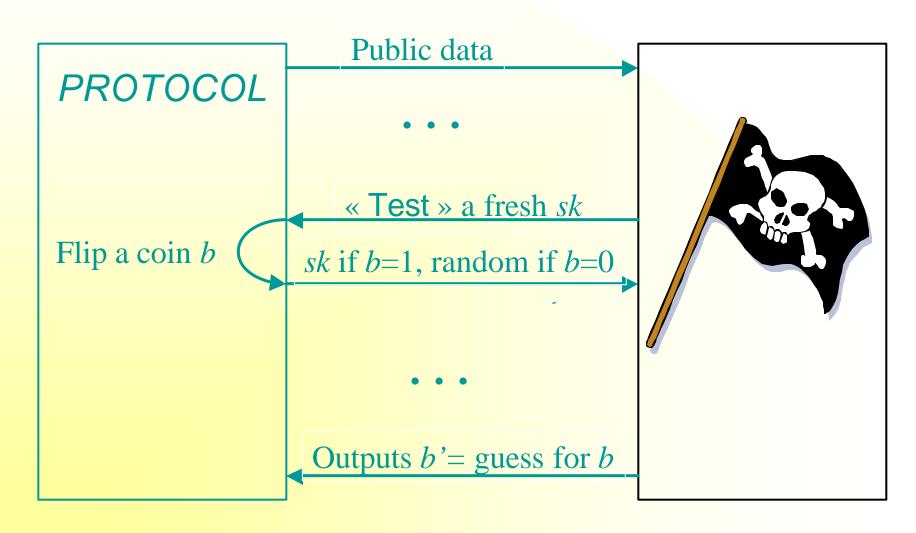


Modeling the Adversary

- Adversary's interacts with the group via queries
 - * setup: initialize the multicast group
 - join/remove: add or remove players to multicast group



Security Definitions (AKE)



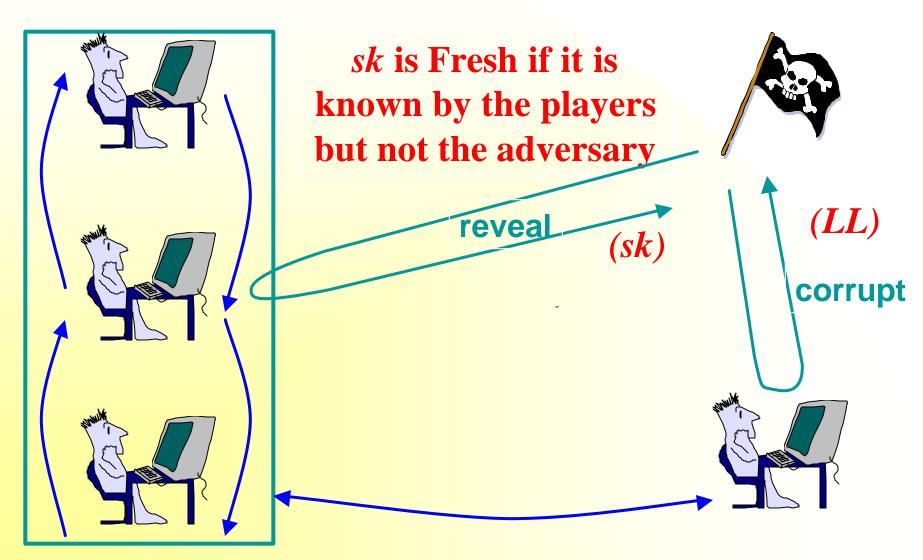
OUR CONTRIBUTIONS

- Concurrent executions considered
- Forward-secrecy
 - * Strong-corruption and weak corruption
- Use of secure crypto-devices
 - Crypto-processor and smart card
- Standard assumptions
 - No random oracle
 - Improved security proof

Forward-Secrecy

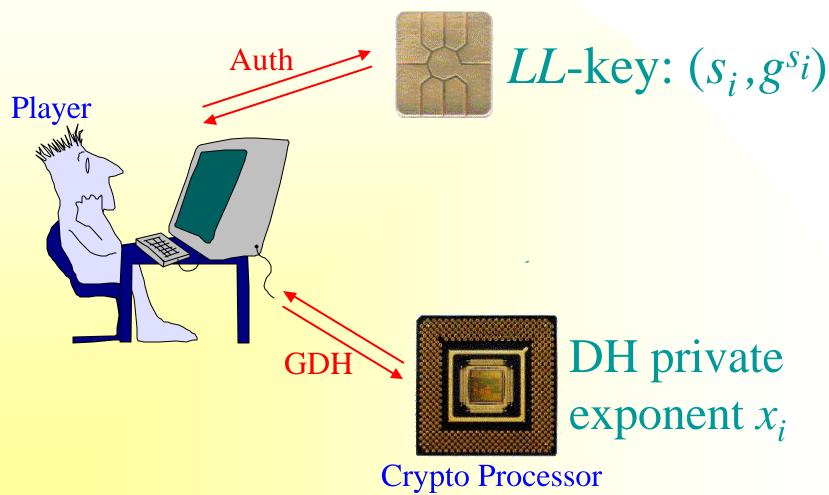
- The corruption of long-lived keys *LL* should not entail the security of *previously* established session keys *sk*.
- 2 flavors of forward-secrecy can be defined:
 - Weak-corruption model: adversary can obtain LL-keys only.
 - * Strong corruption model: adversary may corrupt private exponents as well.

Freshness vs. Corruption Queries



Crypto-Devices



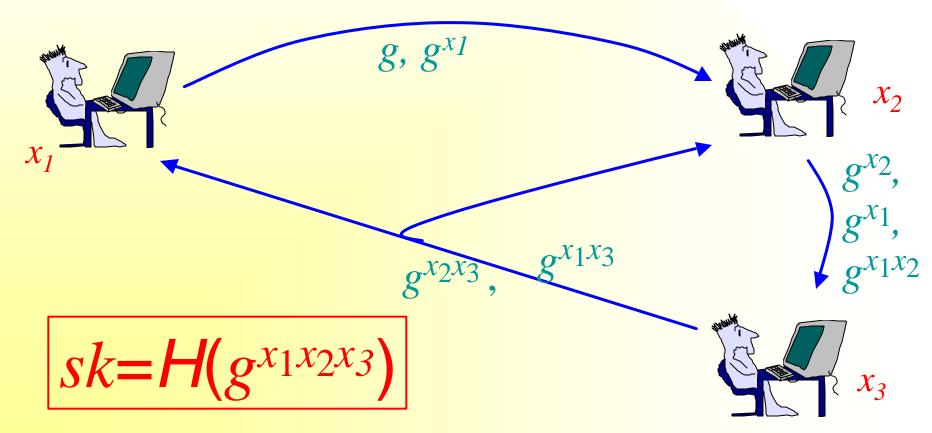


A Secure Authenticated Group Diffie-Hellman Protocol

- The session key is
 - $t sk = H(g^{x_1x_2...x_n})$
- Ring-Based with flows
- Defined by three algorithms
 - * SETUP
 - * REMOVE
 - † JOIN
- Many details abstracted out

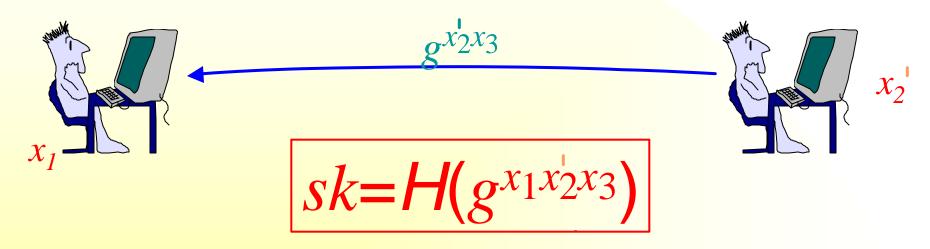
The SETUP Algorithm

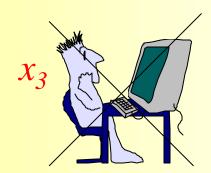
- Up-flow: U_i raises to x_i and forwards to U_{i+1}
- Down-flow: U_n raises to x_n and broadcasts



The REMOVE Algorithm

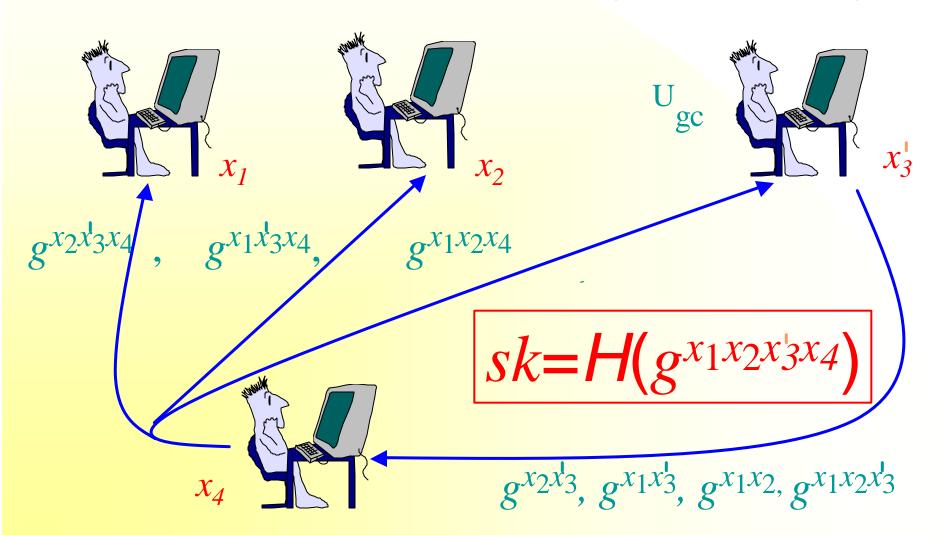
Down-flow of the SETUP algorithm





The JOIN Algorithm

* SETUP initiated by player with highest index in group



Standard assumptions

- Group Decisional Diffie-Hellman
- Multi-Decisional Diffie-Hellman
- Message Authentication Codes (MAC)
- Entropy-smoothing theorem

GDDH Assumptions

Given some subsets of indices in $I=\{1,...,n\}$, and all values:

 $g^{?}i?J^{x}i$, for every given subset J

- † Decide whether a given value is $g^{x_1...x_n}$ or not.
 - * Eg.: Given g^a, g^b, g^{ac} , distinguish g^{abc} from a random value g^r .

Multi-DDH Assumptions

- * Given some n values: $g^{x}i$, for i=1,...n
- \bullet Decide whether n(n-1)/2 values are

$$g^{x_i x_j}$$
 or not.

- * Eg.: Given g^a , g^b , g^c , distinguish g^{ab} , g^{ac} , g^{bc} from three random values g^r , g^s , g^t .
- MDDH problem can easily be reduced to DDH

Message Authentication Codes (MAC)

- * Kgen: outputs a random string k of length l
- Sign/Verif: produces and verifies a MAC from m and k
- MACs will be used to authenticate the flows between players

* MACs exist if OW-functions exist.

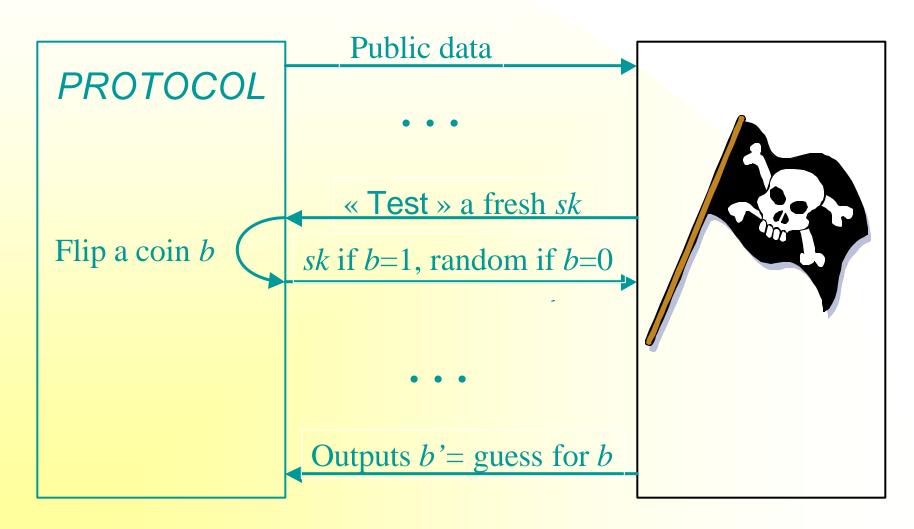
Entropy-smoothing theorem

- Used to derivate keys from g...
- Let D be a distribution of length s and entropy?. Let H be a universal hash function from k-bits x s-bits to l-bits.

Then the following (l+k)-bits distributions are $2^{-(e+1)}$ -statistically close, where l=?-2e:

$$H_r(x)||r|$$
 and $y||r|$
 $x?_D\{0,1\}^s$, Uniform

Security Definitions (AKE)



Security Theorem (AKE)

- Security is measured as the adversary's advantage in guessing the bit b involved in the Test-query
- This advantage is a function of
 - the adversary's advantage in breaking the Group DDH
 - the adversary's advantage in breaking the MAC scheme
 - the adversary's advantage in breaking the Multi-DDH

† Theorem

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Adv<sup>ake</sup>(T,Q,q_s)? 2nQ\cdot \text{Adv}^{\text{gddh}}(T') + n(n-1)\cdot \text{Succ}^{\text{cma}}(T) + 2\cdot \text{Adv}^{\text{mddh}}(T') + \text{w negligible terms} \gg T'? T + nQ\cdot T_{\text{exp}}(k)
```

Sketch of Proof

Game 0: Real attack

Game 1: Abort if a MAC forgery occurs

Game 2: Guess the execution in which the Test-query occurs

Game 3: Simulate the flows from a true GDDH tuple based on the guesses above

Game 4: Simulate the flows, but with a bad GDDH tuple

Game 5: Answer the Test-query at random, letting no advantage to the adversary.

Difference with [A01]

- Group Diffie-Hellman term is relative to the total number of players n (instead of the size of multicast group s)
 - **Loss from** $O(s^3)$ Adv^{ddh} to $O(n^3)$ Adv^{ddh}
- * But:
 - * Improved by a binomial factor from $s\binom{n}{s}$ to n
 - **Better compared to** n^3/s^3
- ? This is a good deal

Conclusion and Future Work

Summary

- a model for strong-corruption
- * a proof allowing for concurrent sessions
- * a proof that does not require an ideal-hash assumption

Work in Progress

* "Group DH Key Exchange secure Against Dictionary Attacks"